

25a) $y' = Ay$, $A = \begin{pmatrix} -1 & 6 \\ -3 & 8 \end{pmatrix}$, $\lambda^2 - 7\lambda + 10 = 0$
eigenvalues and eigenvectors

$\lambda^{(A)} = 5, \lambda^{(B)} = 2$

$\underline{u}_A = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\underline{u}_B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

general solution

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} e^{5x} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C^{(B)} e^{2x} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

b) $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$, $\lambda^2 + 2\lambda = 0$

$\lambda_A = 0, \lambda_B = -2$ $\underline{u}_A = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\underline{u}_B = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C^{(B)} e^{2x} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

c) $A = \begin{pmatrix} -4 & -8 \\ 4 & 4 \end{pmatrix}$, $\lambda^2 + 16 = 0$

$\lambda^{(A)} = 4i, \lambda^{(B)} = -4i$ $\underline{u}_A = \begin{pmatrix} -1+i \\ 1 \end{pmatrix}$, $\underline{u}_B = \begin{pmatrix} -1-i \\ 1 \end{pmatrix}$

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} \begin{pmatrix} -1+i \\ 1 \end{pmatrix} e^{4ix} + C^{(B)} \begin{pmatrix} -1-i \\ 1 \end{pmatrix} e^{-4ix}$

d) $A = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix}$, $\lambda^2 + 2\lambda + 10 = 0$

$\lambda^{(A)} = -1+3i, \lambda^{(B)} = -1-3i$ $\underline{u}_A = \begin{pmatrix} 1 \\ i \end{pmatrix}$, $\underline{u}_B = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} e^{(-1+3i)x} \begin{pmatrix} 1 \\ i \end{pmatrix} + C^{(B)} e^{(-1-3i)x} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

e) $A = \begin{pmatrix} -6 & 4 \\ -8 & 2 \end{pmatrix}$, $\lambda^2 + 4\lambda + 20 = 0$

$\lambda^{(A)} = -2+4i, \lambda^{(B)} = -2-4i$ $\underline{u}_A = \begin{pmatrix} 1 \\ 1+i \end{pmatrix}$, $\underline{u}_B = \begin{pmatrix} 1 \\ 1-i \end{pmatrix}$

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} e^{(-2+4i)x} \begin{pmatrix} 1 \\ 1+i \end{pmatrix} + C^{(B)} e^{(-2-4i)x} \begin{pmatrix} 1 \\ 1-i \end{pmatrix}$

26a) $A = \begin{pmatrix} -8 & -10 \\ -5 & 7 \end{pmatrix}$ $\lambda^{(A)} = 2, \lambda^{(B)} = -3$

$\underline{u}_A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\underline{u}_B = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2x} + C^{(B)} \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-3x}$

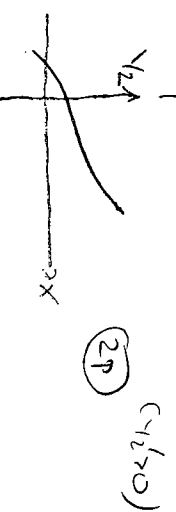
initial conditions

$3 = C^{(A)} + 2C^{(B)}$

$1 = -C^{(A)} - C^{(B)}$

$y_1 = -5e^{2x} + 8e^{-3x}$

$y_2 = 5e^{2x} - 4e^{-3x}$



$$b) \underline{A} = \begin{pmatrix} -6 & 1 \\ 0 & -3 \end{pmatrix} \quad \lambda^{(A)} = -6, \lambda^{(B)} = -3 \quad (2P)$$

$$\underline{u}_A = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \underline{u}_B = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (2P)$$

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} e^{-6x} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C^{(B)} e^{-3x} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$1 = y_1(0) = C^{(A)} + C^{(B)}$$

$$C^{(A)} = 1$$

$$0 = y_2(0) = C^{(B)}$$

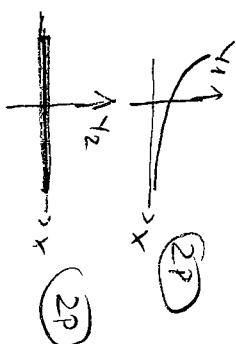
$$(2P) = 0$$

$$C^{(B)} = 0$$

$$(2P)$$

$$y_1 = e^{-6x}$$

$$y_2 = 0$$



$$c) \underline{A} = \begin{pmatrix} -1 & 5 \\ -5 & 1 \end{pmatrix} \quad \lambda^{(A)} = 2\sqrt{6}i \quad \lambda^{(B)} = -2\sqrt{6}i \quad (2P)$$

$$\underline{u}_A = \begin{pmatrix} 5 \\ 1+2\sqrt{6}i \end{pmatrix}, \underline{u}_B = \begin{pmatrix} 5 \\ 1-2\sqrt{6}i \end{pmatrix} \quad (2P)$$

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C^{(A)} e^{2\sqrt{6}ix} \begin{pmatrix} 5 \\ 1+2\sqrt{6}i \end{pmatrix} + C^{(B)} e^{-2\sqrt{6}ix} \begin{pmatrix} 5 \\ 1-2\sqrt{6}i \end{pmatrix} \quad (2P)$$

$$5 = y_1(0) = 5C^{(A)} + 5C^{(B)}$$

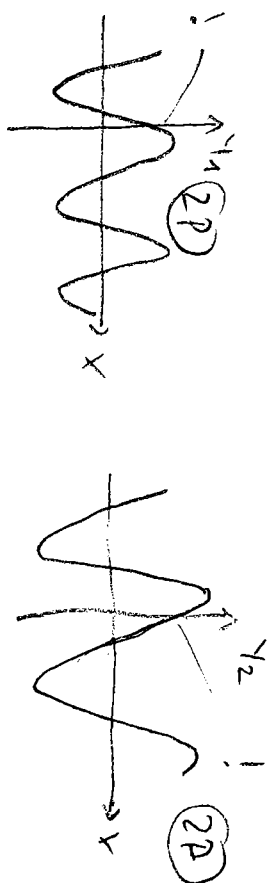
$$(2P)$$

$$5 = y_2(0) = (C^{(A)} + C^{(B)}) + 2\sqrt{6}i(C^{(A)} - C^{(B)})$$

$$= 0 \quad C^{(A)} = \frac{1}{2} - \frac{\sqrt{6}}{6}i, \quad C^{(B)} = \frac{1}{2} + \frac{\sqrt{6}}{6}i \quad (2P)$$

$$y_1 = 5 \cos(2\sqrt{6}x) + \frac{5\sqrt{6}}{3} \sin(2\sqrt{6}x)$$

$$y_2 = 5 \cos(2\sqrt{6}x) + \frac{5\sqrt{6}}{3} \sin(2\sqrt{6}x)$$



$$27a) 9\lambda^2 + 6\lambda + 19 = (3\lambda + 1)^2 + 18 = 0 \quad (2P)$$

$$\Rightarrow \lambda^{(A)} = -\frac{1}{3} + \sqrt{2}i \quad \lambda^{(B)} = -\frac{1}{3} - \sqrt{2}i \quad (2P)$$

$$y = C^{(A)} e^{(-\frac{1}{3} + \sqrt{2}i)x} + C^{(B)} e^{(-\frac{1}{3} - \sqrt{2}i)x} \quad (2P)$$

$$b) y_1 = y, y_2 = y' \quad C: e^{-y_2} = y_1' = -\frac{2}{3}y_1 - \frac{19}{9}y_1$$

$$\Rightarrow y_1' = y_2$$

$$\Rightarrow \underline{A} = \begin{pmatrix} 0 & 1 \\ -\frac{19}{9} & -\frac{2}{3} \end{pmatrix} \quad (2P)$$

$$\Rightarrow \lambda^2 + \frac{2}{3}\lambda + \frac{19}{9} = 0$$

$$(2P)$$

$$\lambda^{(A)} = -\frac{1}{3} + \sqrt{2}i$$

$$\lambda^{(B)} = -\frac{1}{3} - \sqrt{2}i$$

$$\underline{u}_A = \begin{pmatrix} 1 \\ -\frac{1}{3} + \sqrt{2}i \end{pmatrix}, \quad \underline{u}_B = \begin{pmatrix} 1 \\ -\frac{1}{3} - \sqrt{2}i \end{pmatrix} \quad (2P)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = D^{(A)} e^{(-\frac{1}{3} + \sqrt{2}i)x} \begin{pmatrix} 1 \\ -\frac{1}{3} + \sqrt{2}i \end{pmatrix} + D^{(B)} e^{(-\frac{1}{3} - \sqrt{2}i)x} \begin{pmatrix} 1 \\ -\frac{1}{3} - \sqrt{2}i \end{pmatrix} \quad (2P)$$

$$\begin{aligned} c) \quad y &= C^{(A)} e^{(-\frac{1}{3} + \sqrt{2}i)x} + C^{(B)} e^{(-\frac{1}{3} - \sqrt{2}i)x} \\ &= D^{(A)} e^{(-\frac{1}{3} + \sqrt{2}i)x} + D^{(B)} e^{(-\frac{1}{3} - \sqrt{2}i)x} \\ &= y_1 \quad \text{C if we choose } C^{(A/B)} = D^{(A/B)} \end{aligned} \quad (4P)$$

$$\begin{aligned} y' &= C^{(A)} \left(-\frac{1}{3} + \sqrt{2}i\right) e^{(-\frac{1}{3} + \sqrt{2}i)x} + C^{(B)} \left(-\frac{1}{3} - \sqrt{2}i\right) e^{(-\frac{1}{3} - \sqrt{2}i)x} \\ &= y_2 \quad (C^{(A/B)} = D^{(A/B)}) \end{aligned} \quad (4P)$$

$$I) \quad y = y_1, \quad y'' = 2y_1' - y_2' = 2y_1' - y_2'$$

$$\begin{aligned} y_1' &= 2y_1 - y_2 \quad \Rightarrow \quad y_1' = 2y_1 - y_2 \quad \text{Solve for} \\ y_2' &= -y_1 - 3y_2 \quad y_2' = -y_1 - 3y_2 \quad y_2, y_2' \end{aligned}$$

$$\Rightarrow y_2' = -y_1 - 3(2y_1 - y_1') = -7y_1 + 3y_1' \quad (10P)$$

$$\Rightarrow y_1'' = 2y_1' + 7y_1 - 3y_1' \Rightarrow y_1'' + y_1' - 7y_1 = 0 \quad (10P)$$